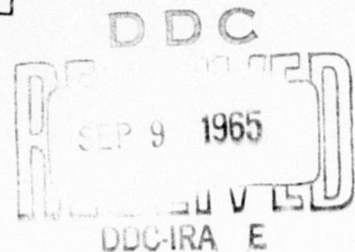# On the Determination of the Deck Motion of Aircraft Carriers

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July 19, 1965

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A method is presented for determining the deck motion of an aircraft carrier in terms of the local reference coordinate system of the ship. Determination of motion is a prelude to prediction, and prediction is required to facilitate safe landing operations under all weather conditions. Both determination and prediction are necessary in terms of the same local reference coordinate system as that used by the landing aircraft themselves.

The finding of the deck motion is deterministic rather than stochastic, inasmuch as the probability of knowing the sea state at any given time and place and the ship's dynamic response thereto is less than the probability of a landing accident not occurring. The motion would be determined by a differential analyzer solving a system of six differential equations of ship motion. Inputs to the differential analyzer are readings of the accelerometers and the latitude and longitude data of the ship. The accelerometers are fixed to the body axes of the vessel, and "stable platforms" are therefore not required. Computation is thus performed only by electronic means.

INTRODUCTION

There are essentially two ways of determining the deck motion of aircraft carriers, and these two methods basically apply to the determination of the motion of any vehicle. One is a self-contained method, such as inertial or geomagnetic, and the other is based on the use of guideposts such as the sun or the other stars. In the guidepost method, sun or stellar trackers would be used. If the trackers were optical, however, the method would be foreclosed by inclement weather. If the trackers were radio telescopes, then the vessel must carry the appropriate antennas which may not be feasible because of their size and dynamics.

One could argue that the deck motion of a vessel would be known if the characteristic behavior pattern of the vessel were known for a set of given surface waveforms of the sea. Such characteristic behavior can be ascertained experimentally, but only for the waveforms used in the experiments, unless the differential equations of the nonlinear dynamic system of ship and sea can be obtained. But this argument would be valid only if we could determine the contemporary surface waveform of the neighboring sea with sufficient precision from the moving vessel itself, which is indeed a very difficult thing to do. The input to the "black box" must be known in order to know its output without measuring that output, even though we assume a linear box and know the characteristics of the box. If we were to map the seven seas to determine the probabilistic relationships between the surface waveforms and the location and time of the year, then, it is argued, the surface waveforms are everywhere and always known. But the probabilities (assuming that the stochastic tests are exhaustive) would undoubtedly not be greater than the probability of a carrier-landing accident not occurring, and a stochastic method of determination and hence prediction would therefore not be as useful as a deterministic one. This analysis therefore chooses a deterministic rather than a stochastic approach to the matter of deck motion.

Of the self-contained methods of deck-motion determination, the conventional inertial method has been chosen, because the technology of its instrumentation is relatively well advanced. The basic equations for deck motion will be set forth. The solutions of these equations will be the six generalized coordinates of the carrier deck. An analysis of the prediction of this motion will follow in a subsequent report. It is the prediction of the generalized coordinates that is of importance in carrier-landing operations, but we must first know what we need to predict.

Previous research* at NRL was concerned with the overall problem of Carrier All-Weather Flying (CAWF), of which the deck-motion problem is a part. Additionally, some measurements have been made of the motion of particular vessels (*e.g.*, USS MACON) at sea. Data previously obtained give an indication of the frequency content of the wave motion of the sea and are useful in the selection of the numerical parameters of predictors.

DIFFERENTIAL EQUATIONS OF DECK MOTION

An inertial system of motion determination requires a geocentric inertial reference frame, but the actual carrier landing operation is performed in the context of a local Euclidean coordinate system originating at the aircraft carrier and having the local vertical and the two local horizontals as its coordinate axes. We therefore require two reference coordinate systems, as well as several other coordinate systems.

Figure 1 shows what appear to be two sets of Euclidean coordinate systems. One set originates at the geocenter, and the other set appears to originate at the aircraft carrier. The set apparently originating at the carrier, however, consists of coordinate lines through the carrier that are actually part of the set originating at the geocenter. Coordinate system υ is the inertial reference frame. The coordinate axes of υ that pass through the vessel's center are called x^i. The local vertical at the ship is axis z^1; local horizontal axis z^2 is tangent to the latitude curve of the earth; local horizontal axis z^3 is tangent to the longitude curve of the earth. The z^i coordinate axes are those coordinate lines of the geocenter-originating η coordinate system that pass through the ship's center. Thus η rotates about the geocenter as the ship moves over the surface of th · earth.

The y^i axes are the ship's body axes. Axis y^1 is the ship's vertical axis, and the y^1 plane is therefore the plane of the flight deck; y^2 is the fore-and-aft axis; and y^3 is the athwartship axis. These axes are the coordinate lines through the ship's center of the w coordinate system originating at
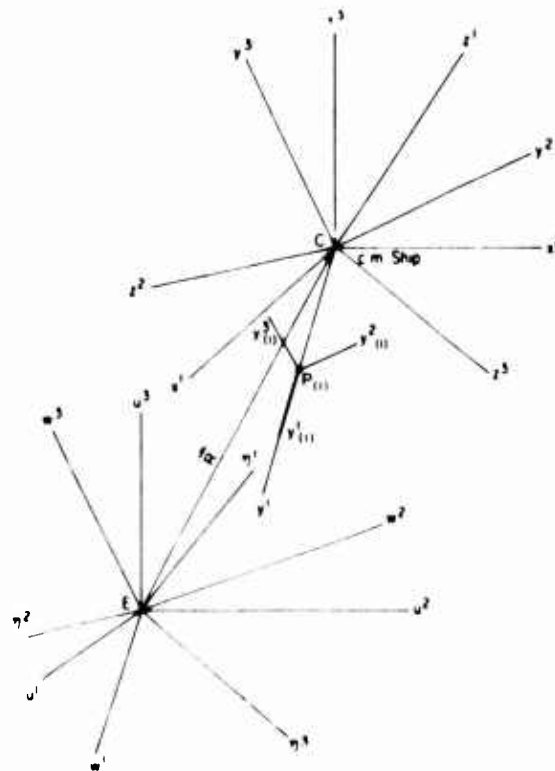

Fig 1 — Geocentric and local ship's coordinate systems

the geocenter. The **w** coordinate system rotates about the geocenter as the ship rolls, yaws, and pitches about its center of mass.

The motion of the ship is established if we determine the motion of three noncolinear points in the ship. We will choose a point on each of the body axes, and will fix our accelerometers directly to the body axes, thus removing any need for "stable platforms." At each point $P_{(a)}$ on body axis y^a, and at distance $\lambda_{(a)}$ from the center of mass of the ship, we will establish a coordinate system $y_{(a)}$ such that each axis $y^i_{(a)}$ is respectively parallel to y^i. The accelerometers are mounted at each point such that the axes of the instruments are respectively parallel to the body axes of the ship. Since, however, the ship has six degrees of freedom and therefore six generalized coordinates, only six, rather than nine, accelerometers need be used. At $P_{(1)}$, we will mount two accelerometers with axes respectively parallel to y^2 and y^3; at $P_{(2)}$, we will mount two accelerometers with axes respectively parallel to y^1 and y^3; and at $P_{(3)}$, we will mount two accelerometers respectively parallel to y^1 and y^2.

We will let the $\mathbf{e}_i$ be the base vectors of the **u** coordinate system and therefore of **x** as well. The $\mathbf{a}_i$ are the base vectors of the **w** coordinate system and therefore also of **y**, and the $\mathbf{b}_i$ are the base vectors of the $\boldsymbol{\eta}$ coordinate system and therefore also of **z**.

The position vector of the center of mass of the ship is **R**, and the position vectors of the accelerometer points $P_{(a)}$ are respectively $\mathbf{r}_{(a)}$. We therefore have

$$\mathbf{r}_{(a)} = \mathbf{R} + \lambda_{(a)} \mathbf{a}_a \tag{1}$$

and the acceleration of point $P_{(a)}$ is therefore

$$\ddot{\mathbf{r}}_{(a)} = \ddot{\mathbf{R}} + \lambda_{(a)} \ddot{\mathbf{a}}_a. \tag{2}$$

In terms of the coordinates of point $P_{(a)}$ in the **w** coordinate system (which is essentially the body coordinate system), the position vector is

$$\mathbf{r}_{(a)} = \mathbf{a}_i w^i_{(c)} + \lambda_{(a)} \mathbf{a}_a = \mathbf{a}_i (w^i_{(c)} + \lambda_{(a)} \delta^i_a) \tag{3}$$

where the $w^i_{(c)}$ are the coordinates of the ship's center. The velocity of $P_{(a)}$ is therefore

$$\dot{\mathbf{r}}_{(a)} = \dot{\mathbf{a}}_i w^i_{(c)} + \mathbf{a}_i \dot{w}^i_{(c)} + \lambda_{(a)} \dot{\mathbf{a}}_a = \dot{\mathbf{a}}_i (w^i_{(c)} + \lambda_{(a)} \delta^i_a) + \mathbf{a}_i \dot{w}^i_{(c)} \tag{4}$$

and the acceleration is

$$\ddot{\mathbf{r}}_{(a)} = \ddot{\mathbf{a}}_i w^i_{(c)} + 2\dot{\mathbf{a}}_i \dot{w}^i_{(c)} + \mathbf{a}_i \ddot{w}^i_{(c)} + \lambda_{(a)} \ddot{\mathbf{a}}_a = \ddot{\mathbf{a}}_i (w^i_{(c)} + \lambda_{(a)} \delta^i_a) + 2\dot{\mathbf{a}}_i \dot{w}^i_{(c)} + \mathbf{a}_i \ddot{w}^i_{(c)}. \tag{5}$$

But

$$\mathbf{a}_i = \mathbf{e}_j \frac{\partial u^j}{\partial w^i} \tag{6}$$

where the $\mathbf{e}_j$ are constant, and hence

$$\dot{\mathbf{a}}_i = \mathbf{e}_j \frac{d}{dt} \frac{\partial u^j}{\partial w^i} \tag{7}$$

and

$$\ddot{\mathbf{a}}_i = \mathbf{e}_j \frac{d^2}{dt^2} \frac{\partial u^j}{\partial w^i}. \tag{8}$$

Since

$$\mathbf{e}_j = \mathbf{a}_k \frac{\partial w^k}{\partial u^j} \tag{9}$$

we obtain

$$\dot{\mathbf{a}}_i = \mathbf{a}_k \frac{\partial w^k}{\partial u^j} \frac{d}{dt} \frac{\partial u^j}{\partial w^i} \tag{10}$$

and

$$\ddot{\mathbf{a}}_i = \mathbf{a}_k \frac{\partial w^k}{\partial u^j} \frac{d^2}{dt^2} \frac{\partial u^j}{\partial w^i}. \tag{11}$$

In a carrier-landing operation, we are interested in the motion of the carrier in the context of the η coordinate system rather than of the u coordinate system. We therefore write

$$\frac{\partial w^k}{\partial u^j} = \frac{\partial w^k}{\partial \eta^m} \frac{\partial \eta^m}{\partial u^j} \tag{12}$$

and

$$\frac{\partial u^j}{\partial w^i} = \frac{\partial u^j}{\partial \eta^n} \frac{\partial \eta^n}{\partial w^i}. \tag{13}$$

Thus

$$\frac{d}{dt} \frac{\partial u^j}{\partial w^i} = \frac{\partial u^j}{\partial \eta^n} \frac{d}{dt} \frac{\partial \eta^n}{\partial w^i} + \frac{\partial \eta^n}{\partial w^i} \frac{d}{dt} \frac{\partial u^j}{\partial \eta^n} \tag{14}$$

and

$$\frac{d^2}{dt^2} \frac{\partial u^j}{\partial w^i} = \frac{\partial u^j}{\partial \eta^n} \frac{d^2}{dt^2} \frac{\partial \eta^n}{\partial w^i} + 2 \frac{d}{dt} \frac{\partial u^j}{\partial \eta^n} \frac{d}{dt} \frac{\partial \eta^n}{\partial w^i} + \frac{\partial \eta^n}{\partial w^i} \frac{d^2}{dt^2} \frac{\partial u^j}{\partial \eta^n}. \tag{15}$$

The sets of equations of transformation between w and η and between η and u are linear, and therefore $\partial w^k/\partial \eta^m$ and $\partial \eta^n/\partial u^j$ are independent of the coordinates themselves and are functions of the angles describing one coordinate system in terms of the other. The angles between w and η are their set of Eulerian angles, and the angles between η and u are the earth's longitude and latitude of the position of the ship. The Eulerian angles between w and η define the roll, pitch, and yaw of the ship about its center with respect to the local reference coordinate system z. They are therefore three of the six generalized coordinates of the ship. The remaining three generalized coordinates are the coordinates $\eta^i_{(c)}$ of the ship's center, essentially in its own local reference coordinate system.

Let the $\phi^i(t)$, $i = 1,2,3$, be the Eulerian angles. And let ξ be a spherical coordinate system, shown in Fig. 2, originating at the geocenter; ξ^2 is the longitude relative to the nonrotating u coordinate system, and ξ^3 is the latitude, of the vessel's position on the surface of the earth.

We obtain

$$\frac{d}{dt} \frac{\partial \eta^n}{\partial w^i} = \frac{\partial^2 \eta^n}{\partial w^i \partial \phi^\mu} \dot{\phi}^\mu \tag{16}$$

and

$$\frac{d}{dt} \frac{\partial u^j}{\partial \eta^n} = \frac{\partial^2 u^j}{\partial \eta^n \partial \xi^r} \dot{\xi}^r. \tag{17}$$
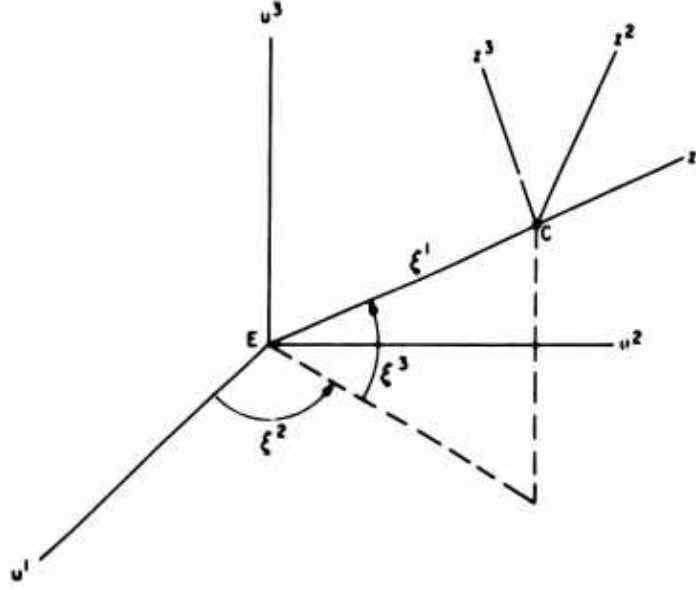
Fig. 2 — Spherical coordinate system
for the ship's location

Therefore,

$$\frac{d^2}{dt^2}\frac{\partial \eta^n}{\partial w^i} = \frac{d}{dt}\left(\frac{\partial^2 \eta^n}{\partial w^i \, \partial \phi^p}\dot{\phi}^p\right) = \frac{\partial^3 \eta^n}{\partial w^i \, \partial \phi^p \, \partial \phi^q}\dot{\phi}^p\dot{\phi}^q + \frac{\partial^2 \eta^n}{\partial w^i \, \partial \phi^p}\ddot{\phi}^p \tag{18}$$

and

$$\frac{d^2}{dt^2}\frac{\partial u^j}{\partial \eta^n} = \frac{d}{d}\left(\frac{\partial^2 u^j}{\partial \eta^n \, \partial \xi^r}\dot{\xi}^r\right) = \frac{\partial^3 u^j}{\partial \eta^n \, \partial \xi^r \, \partial \xi^s}\dot{\xi}^r\dot{\xi}^s + \frac{\partial^2 u^j}{\partial \eta^n \, \partial \xi^r}\ddot{\xi}^r. \tag{19}$$

Substituting Eqs. (16) through (19) in Eqs. (14) and (15), we obtain

$$\frac{d}{dt}\frac{\partial u^j}{\partial w^i} = \frac{\partial u^j}{\partial \eta^n}\frac{\partial^2 \eta^n}{\partial w^i \, \partial \phi^p}\dot{\phi}^p + \frac{\partial \eta^n}{\partial w^i}\frac{\partial^2 u^j}{\partial \eta^n \, \partial \xi^r}\dot{\xi}^r \tag{20}$$

and

$$\frac{d^2}{dt^2}\frac{\partial u^j}{\partial w^i} = \frac{\partial u^j}{\partial \eta^n}\left(\frac{\partial^3 \eta^n}{\partial w^i \, \partial \phi^p \, \partial \phi^q}\dot{\phi}^p\dot{\phi}^q + \frac{\partial^2 \eta^n}{\partial w^i \, \partial \phi^p}\ddot{\phi}^p\right) + \frac{\partial \eta^n}{\partial w^i}\left(\frac{\partial^3 u^j}{\partial \eta^n \, \partial \xi^r \, \partial \xi^s}\dot{\xi}^r\dot{\xi}^s + \frac{\partial^2 u^j}{\partial \eta^n \, \partial \xi^r}\ddot{\xi}^r\right). \tag{21}$$

The time derivatives of the base vectors can now be written as

$$\dot{\mathbf{a}}_i = \mathbf{a}_k\frac{\partial w^k}{\partial u^j}\frac{d}{dt}\frac{\partial u^j}{\partial w^i} = \mathbf{a}_k\frac{\partial w^k}{\partial \eta^m}\frac{\partial \eta^m}{\partial u^j}\left(\frac{\partial u^j}{\partial \eta^n}\frac{\partial^2 \eta^n}{\partial v^i \, \partial \phi^p}\dot{\phi}^p + \frac{\partial \eta^n}{\partial w^i}\frac{\partial^2 u^j}{\partial \eta^n \, \partial \xi^r}\dot{\xi}^r\right) \tag{22}$$

and

$$\ddot{\mathbf{a}}_i = \mathbf{a}_k\frac{\partial w^k}{\partial u^j}\frac{d^2}{dt^2}\frac{\partial u^j}{\partial w^i} = \mathbf{a}_k\frac{\partial w^k}{\partial \eta^m}\frac{\partial \eta^m}{\partial u^j}\left[\frac{\partial u^j}{\partial \eta^n}\left(\frac{\partial^3 \eta^n}{\partial w^i \, \partial \phi^p \, \partial \phi^q}\dot{\phi}^p\dot{\phi}^p + \frac{\partial^2 \eta^n}{\partial w^i \, \partial \phi^p}\ddot{\phi}^p\right)\right.$$

$$\left. + \frac{\partial \eta^n}{\partial w^i}\left(\frac{\partial^3 u^j}{\partial \eta^n \, \partial \xi^r \, \partial \xi^s}\dot{\xi}^r\dot{\xi}^s + \frac{\partial^2 u^j}{\partial \eta^n \, \partial \xi^r}\ddot{\xi}^r\right)\right]. \tag{23}$$

Substituting Eqs. (22) and (23) in Eq. (5), we obtain

$$\ddot{\mathbf{r}}_{(a)} = \mathbf{a}_k \, \ddot{u}_{(c)}^a + 2\mathbf{a}_k \frac{\partial w^k}{\partial \eta^m} \frac{\partial \eta^m}{\partial u^j} \left(\frac{\partial u^j}{\partial \eta^n} \frac{\partial^2 \eta^n}{\partial w^i \, \partial \phi^p} \dot{\phi}^p + \frac{\partial \eta^n}{\partial w^i} \frac{\partial^2 u^j}{\partial \eta^n \, \partial \xi^r} \dot{\xi}^r \right) \dot{w}_{(c)}^i$$

$$+ \mathbf{a}_k \frac{\partial w^k}{\partial \eta^m} \frac{\partial \eta^m}{\partial u^j} \left[\frac{\partial u^j}{\partial \eta^n} \left(\frac{\partial^3 \eta^n}{\partial w^i \, \partial \phi^p \, \partial \phi^q} \dot{\phi}^p \dot{\phi}^q + \frac{\partial^2 \eta^n}{\partial w^i \, \partial \phi^p} \ddot{\phi} \right) + \frac{\partial \eta^n}{\partial w^i} \left(\frac{\partial^3 u^j}{\partial \eta^n \, \partial \xi^r \, \partial \xi^s} \dot{\xi}^r \dot{\xi}^s \right. \right.$$

$$\left. \left. + \frac{\partial^2 u^j}{\partial \eta^n \, \partial \xi} \ddot{\xi}^r \right) \right] (w_{(c)}^i + \lambda_{(a)} \delta_a^i). \tag{24}$$

Since

$$w_{(c)}^i + \lambda_{(a)} \delta_a^i \cong u_{(c)}^i \tag{25}$$

we can neglect the last term of Eq. (24), and the components of $\ddot{\mathbf{r}}_{(a)}$ are

$$A_{(a)}^k = \ddot{w}_{(c)}^k + 2 \frac{\partial w^k}{\partial \eta^m} \frac{\partial \eta^m}{\partial u^j} \left(\frac{\partial u^j}{\partial \eta^n} \frac{\partial^2 \eta^n}{\partial w^i \, \partial \phi^p} \dot{\phi}^p + \frac{\partial \eta^n}{\partial w^i} \frac{\partial^2 u^j}{\partial \eta^n \, \partial \xi^r} \dot{\xi}^r \right) \dot{w}_{(c)}^i$$

$$+ \frac{\partial w^k}{\partial \eta^m} \frac{\partial \eta^m}{\partial u^j} \left[\frac{\partial u^j}{\partial \eta^n} \left(\frac{\partial^3 \eta^n}{\partial w^i \, \partial \phi^p \, \partial \phi^q} \dot{\phi}^p \dot{\phi}^q + \frac{\partial^2 \eta^n}{\partial w^i \, \partial \phi^p} \ddot{\phi}^p \right) + \frac{\partial \eta^n}{\partial w^i} \left(\frac{\partial^3 u^j}{\partial \eta^n \, \partial \xi^r \, \partial \xi^s} \dot{\xi}^r \dot{\xi}^s \right. \right.$$

$$\left. \left. + \frac{\partial^2 u^j}{\partial \eta^n \, \partial \xi^r} \ddot{\xi}^r \right) \right] w_{(c)}^i. \tag{26}$$

The actual readings of the accelerometers are

$$h_{(a)}^k = A_{(a)}^k - g^k \tag{27}$$

where the g^k are the components in the $\mathbf{w}$ coordinate system of the gravitational field intensity $\mathbf{g}$. Since components of $\mathbf{g}$ are usually given in η, we write

$$g^k(w) = \gamma^i(\eta) \frac{\partial w^k}{\partial \eta^i} \tag{28}$$

where the γ^i are the components of $\mathbf{g}$ in η.

Equations (27) are the system of six differential equations of motion of the carrier deck; $k, \alpha = 1,2,3$, but $\alpha \neq k$. The solutions of this system of differential equations are the three $w_{(c)}^i(t)$ and the three $\phi^i(t)$. The $h_{(a)}^k(t)$ are the forcing functions of the system.

The $\phi^i(t)$ are three of the generalized coordinates of the carrier deck. The remaining three generalized coordinates are

$$\eta_{(c)}^i = \eta_{(c)}^i (w_{(c)}^1, w_{(c)}^2, w_{(c)}^3; \phi^1, \phi^2, \phi^3) \tag{29}$$

which are functions of the $w_{(c)}^i$ and of the ϕ^i.

EQUATIONS OF COORDINATE TRANSFORMATION

The orthogonal transformations give rise to linear equations of coordinate transformation. In order to obtain the equations of coordinate transformation between $\mathbf{w}$ and η, we cause η to undergo three successive finite rotational displacements ϕ^i. The first rotation is that of η about
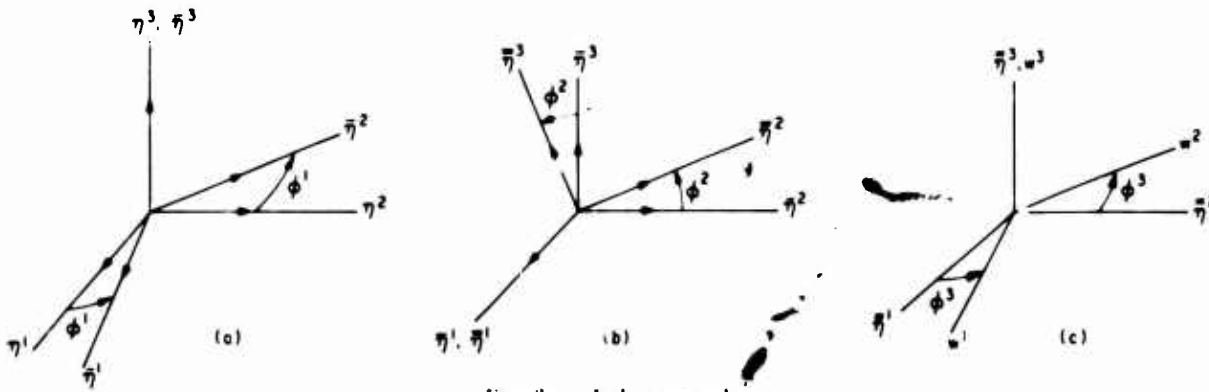
Fig. 3 — Eulerian angles

axis η^3 to obtain another Euclidean coordinate system $\overline{\eta}$, as shown in Fig. 3a, whose base vectors are c_i. This first rotational displacement is the first Eulerian angle ϕ^1. The equations of coordinate transformation between $\overline{\eta}$ and η are

$$\overline{\eta}^i = A^i_j \, \eta^j \tag{30}$$

where

$$(A^i_j) = \begin{pmatrix} \cos \phi^1 & \sin \phi^1 & 0 \\ -\sin \phi^1 & \cos \phi^1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{31}$$

The second Eulerian angle is the finite rotational displacement ϕ^2 of $\overline{\eta}$ about axis $\overline{\eta}^1$ to obtain the Euclidean coordinate system $\overline{\overline{\eta}}$ (Fig. 3b), whose base vectors are d_i. The equations of coordinate transformation are

$$\overline{\overline{\eta}}^i = B^i_j \, \overline{\eta}^j \tag{32}$$

where

$$(B^i_j) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi^2 & \sin \phi^2 \\ 0 & -\sin \phi^2 & \cos \phi^2 \end{pmatrix}. \tag{33}$$

The third Eulerian angle is the finite rotational displacement ϕ^3 of $\overline{\overline{\eta}}$ about $\overline{\overline{\eta}}^3$ to obtain the Euclidean coordinate system w (Fig. 3c), whose base vectors are the a_i. The equations of coordinate transformation are

$$w^i = C^i_j \, \overline{\overline{\eta}}^j \tag{34}$$

where

$$(C^i_j) = \begin{pmatrix} \cos \phi^3 & \sin \phi^3 & 0 \\ -\sin \phi^3 & \cos \phi^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{35}$$

The overall equations of coordinate transformation between w and η are therefore

$$w^i = \Gamma^i_j \, \eta^j \tag{36}$$

where

$$\Gamma^i_j = C_k B^k_m A^m_j \qquad (37)$$

and therefore

$$(\Gamma^i_j) = \begin{pmatrix} \cos\phi^1\cos\phi^3 & \cos\phi^3\sin\phi^1 & \sin\phi^2\sin\phi^3 \\ -\cos\phi^2\sin\phi^1\sin\phi^3 & +\cos\phi^2\cos\phi^1\sin\phi^3 & \\ -\sin\phi^3\cos\phi^1 & -\sin\phi^3\sin\phi^1 & \cos\phi^3\sin\phi^2 \\ -\cos\phi^2\sin\phi^1\cos\phi^3 & +\cos\phi^2\cos\phi^1\cos\phi^3 & \\ \sin\phi^1\sin\phi^2 & -\sin\phi^2\cos\phi^1 & \cos\phi^2 \end{pmatrix}. \qquad (38)$$

The inverse equations of coordinate transformation are

$$\eta^i = E^i_j w^j \qquad (39)$$

where the matrix (E^i_j) is the inverse of (Γ^i_j) and is therefore

$$(E^i_j) = \begin{pmatrix} \cos\phi^1\cos\phi^3 & -\sin\phi^3\cos\phi^1 & \sin\phi^1\sin\phi^2 \\ -\cos\phi^2\sin\phi^1\sin\phi^3 & -\cos\phi^2\sin\phi^1\cos\phi^3 & \\ \cos\phi^3\sin\phi^1 & -\sin\phi^3\sin\phi^1 & -\sin\phi^2\cos\phi^1 \\ +\cos\phi^2\cos\phi^1\sin\phi^3 & +\cos\phi^2\cos\phi^1\cos\phi^3 & \\ \sin\phi^2\sin\phi^3 & \cos\phi^3\sin\phi^2 & \cos\phi^2 \end{pmatrix}. \qquad (40)$$

The transformation $\eta \leftrightarrow u$ is also linear and involves the longitude and latitude angles, ξ^2 and ξ^3, respectively. In order to derive these equations of coordinate transformation, we at first consider the finite rotational displacement of u about u^3 through a finite angle ξ^2, resulting therefore in a coordinate system ψ (Fig. 4). This first orthogonal transformation is

$$\psi^1 = u^1\cos\xi^2 + u^2\sin\xi^2$$

$$\psi^2 = -u^1\sin\xi^2 + u^2\cos\xi^2 \qquad (41)$$

$$\psi^3 = u^3.$$

We then rotate ψ through a finite angle ξ^3 about axis ψ^2, resulting therefore in coordinate system η, as shown in Fig. 5. This second orthogonal transformation is

$$\eta^1 = \psi^1\cos\xi^3 + \psi^3\sin\xi^3$$

$$\eta^2 = \psi^2 \qquad (42)$$

$$\eta^3 = -\psi^1\sin\xi^3 + \psi^3\cos\xi^3.$$

The overall equations of coordinate transformation are
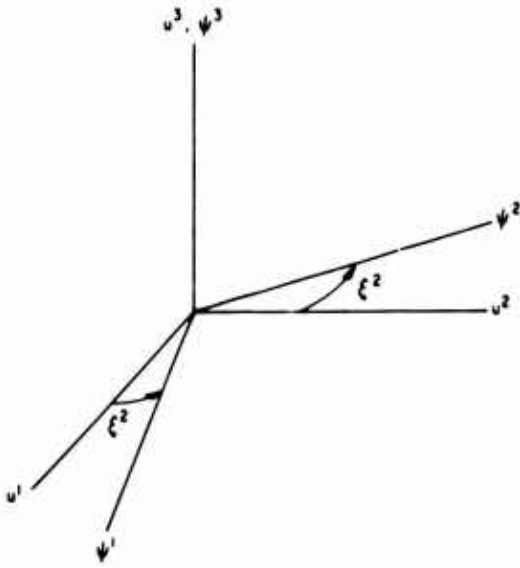
$$\eta^i = H^i_j w^j \qquad (43)$$

Fig. 4 — Orthogonal transformation
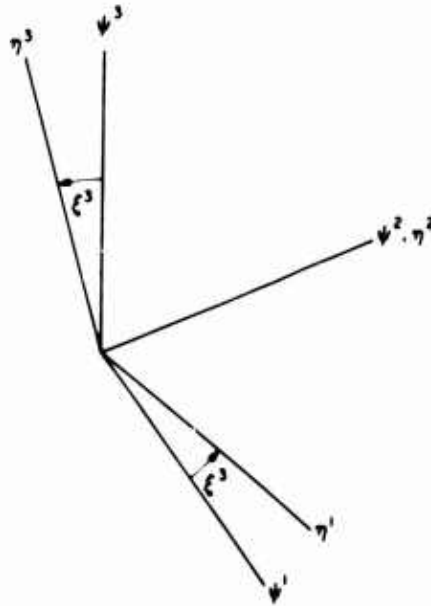due to longitude

Fig. 5 — Orthogonal transformation
due to latitude

where the H_j^i are

$$(H_j^i) = \begin{pmatrix} \cos \xi^2 \cos \xi^3 & \sin \xi^2 & \cos \xi^3 \sin \xi^3 \\ -\sin \xi^2 & \cos \xi^2 & 0 \\ -\sin \xi^3 \cos \xi^2 & -\sin \xi^2 & \sin \xi^3 \cos \xi^3 \end{pmatrix} \qquad (44)$$

The inverse equations of coordinate transformation are

$$u^i = K_j^i \, \eta^i \qquad (45)$$

where the matrix (K_j^i) is the inverse of (H_j^i), and is therefore

$$(K_j^i) = \begin{pmatrix} \cos \xi^2 \cos \xi^3 & -\sin \xi^2 & -\sin \xi^3 \cos \xi^2 \\ \sin \xi^2 \cos \xi^3 & \cos \xi^2 & -\sin \xi^2 \sin \xi^3 \\ \sin \xi^3 & 0 & \cos \xi^3 \end{pmatrix} \qquad (46)$$

We therefore have

$$\frac{\partial w^i}{\partial \eta^j} = \Gamma_j^i$$

$$\frac{\partial \eta^i}{\partial w^j} = E_j^i$$

$$\frac{\partial \eta^i}{\partial u^j} = H_j^i \qquad (47)$$

$$\frac{\partial u^i}{\partial \eta^j} = K_j^i.$$

Upon substitution of Eqs. (26) and (28) in Eq. (27), we obtain the differential equations of motion of the carrier deck as

$$
\ddot{w}^k_{(c)} + 2\frac{\partial w^k}{\partial \eta^m}\frac{\partial \eta^m}{\partial u^j}\left(\frac{\partial u^j}{\partial \eta^n}\frac{\partial^2 \eta^n}{\partial u^i \partial \phi^p}\dot{\phi}^p + \frac{\partial \eta^n}{\partial u^i}\frac{\partial^2 u^j}{\partial \eta^n \partial \xi^r}\dot{\xi}^r\right)\dot{w}^i_{(c)} + \frac{\partial w^k}{\partial \eta^m}\frac{\partial \eta^m}{\partial u^j}\left[\frac{\partial u^j}{\partial \eta^n}\left(\frac{\partial^3 \eta^n}{\partial u^i \partial \phi^p \partial \phi^q}\dot{\phi}^p\dot{\phi}^q\right.\right.
$$

$$
\left.\left. + \frac{\partial^2 \eta^n}{\partial u^i \partial \phi^p}\ddot{\phi}^p\right) + \frac{\partial \eta^n}{\partial u^i}\left(\frac{\partial^3 u^j}{\partial \eta^n \partial \xi^r \partial \xi^s}\dot{\xi}^r\dot{\xi}^s + \frac{\partial^2 u^j}{\partial \eta^n \partial \xi^r}\ddot{\xi}^r\right)\right]w^i_{(c)} - \gamma^i\frac{\partial w^k}{\partial \eta^i} = h^k_{(a)}(t)
$$

$$
(k\ ,x = 1,2,3;\ \alpha \neq k) \tag{48}
$$

where the ξ^2 and ξ^3 give rise to coefficients that are also explicitly dependent on time.

CONCLUSIONS

This report has presented an analysis of the deck motion of an aircraft carrier and has derived the system of differential equations of motion. Motion is ascertained by the solution of the system based on accelerometer readings and on latitude and longitude data as inputs to the system. The accelerometers would be attached to the body axes of the vessel, and therefore no "stable platforms" would be required. All computation would therefore be by electronic means alone.

Current information on the generalized coordinates of motion is required in order that the coordinates be predictable in time. Thus the outputs of the differential analyzer solving the system of differential equations become the inputs to a six-channel predictor.

Motion determination is based on motion measurement rather than on the dynamic response of the vessel to a given sea state; the sea state cannot be determined with sufficiency from the moving vessel itself, and the behavior characteristics of the ship cannot be adequately pre-determined owing to the nonlinearity of the dynamic system of sea and ship.

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DOCUMENT CONTROL DATA · R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author)	2a REPORT SECURITY CLASSIFICATION
U.S. Naval Research Laboratory	UNCLASSIFIED
Washington, D.C. 20390	2b GROUP

3 REPORT TITLE

On the Determination of the Deck Motion of Aircraft Carriers

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

An interim report on one phase of the problem.

5 AUTHOR(S) (Last name, first name, initial)

Crafton, Paul A.

6 REPORT DATE	7a TOTAL NO OF PAGES	7b NO OF REFS
July 19, 1965	12	

8a CONTRACT OR GRANT NO	9a ORIGINATOR'S REPORT NUMBER(S)
NRL Problem R04-04	
b PROJECT NO	NRL Report 6215
RF 002-06-41-4200	
c	9b OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d	

10 AVAILABILITY/LIMITATION NOTICES

Unlimited availability
Available at CFSTI —

11 SUPPLEMENTARY NOTES	12 SPONSORING MILITARY ACTIVITY
	Dept. of the Navy
	(Office of Naval Research)

13 ABSTRACT

A method is presented for determining the deck motion of an aircraft carrier in terms of the local reference coordinate system of the ship. Determination of motion is a prelude to prediction, and prediction is required to facilitate safe landing operations under all weather conditions. Both determination and prediction are necessary in terms of the same local reference coordinate system as that used by the landing aircraft themselves.

The finding of the deck motion is deterministic rather than stochastic, inasmuch as the probability of knowing the sea state at any given time and place and the ship's dynamic response thereto is less than the probability of a landing accident not occurring. The motion would be determined by a differential analyzer solving a system of six differential equations of ship motion. Inputs to the differential analyzer are readings of the accelerometers and the latitude and longitude data of the ship. The accelerometers are fixed to the body axes of the vessel, and "stable platforms" are therefore not required. Computation is thus performed only by electronic means.

DD FORM 1473
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11

14 KEY WORDS	LINK A		LINK B		LINK C	
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